

# Phytoplankton, Microcosms, ODE's, R.

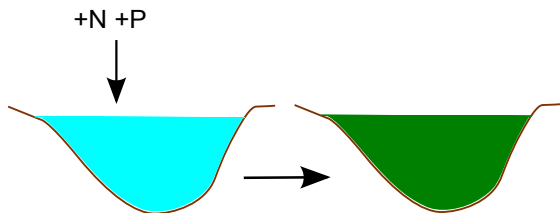
Andrew M. Dolman

useR-Aalborg 01 July 2015

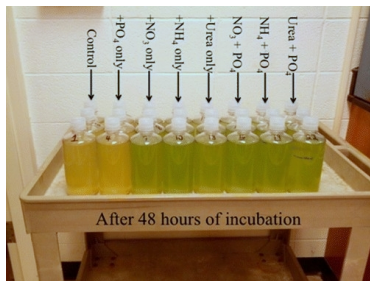
# Ecology



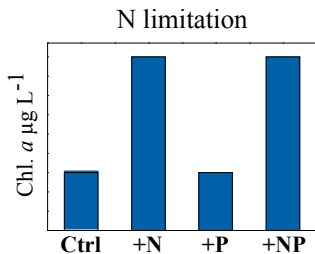
# Eutrophication



## Bioassays

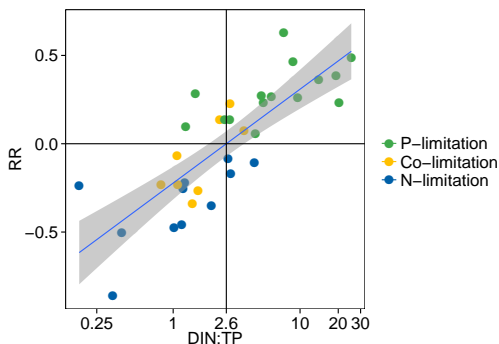


Davis et al (2015)



Kolzau et al (2014)

## Standard statistics



Kolzau et al (2014)

$$RR = \log(P_{response}/N_{response})$$

# ODE's

## Simulate the microcosm with ODE's



Simulate the microcosm with ODE's

## Phytoplankton ( $C_{phy}$ ) + 1 resource ( $N_{phy}$ and $DIN$ )

$$\frac{dC_{phy}}{dt} = \frac{C_{phy} IN_{max} \left(1 - \frac{C_{phy} IN_{min}}{N_{phy}}\right)}{IN_{max} - IN_{min}}$$

$$\frac{dN_{phy}}{dt} = \frac{C_{phy} \left( IN_{max} - \frac{N_{phy}}{C_{phy}} \right) N_{up_{max}} DIN}{(IN_{max} - IN_{min}) (DIN + K_{Nup})}$$

$$\frac{dDIN}{dt} = - \left\{ \frac{C_{phy} \left( IN_{max} - \frac{N_{phy}}{C_{phy}} \right) N_{up_{max}} DIN}{(IN_{max} - IN_{min}) (DIN + K_{Nup})} \right\}$$

Use numerical solvers to step forward in time from initial conditions.

# Technical challenges

- ▶ No standard implementation
  - ▶ Sharing (communication between modellers) is difficult
  - ▶ Re-use is difficult
  - ▶ No standard = hard to get started
- ▶ Theory and code entangled
  - ▶ Poor documentation
  - ▶ Collaboration with non-modellers is difficult

# rodeo

## Definition in tables + Code generation

# rodeo - David Kneis



<https://github.com/dkneis/rodeo>

# Identifiers

	name	type	unit	description	tex	html
1	C_phy	v	$\mu\text{gC/L}$	Cellular C	$C_{\{\text{phy}\}}$	C_phy
2	N_phy	v	$\mu\text{gN/L}$	Cellular N	$N_{\{\text{phy}\}}$	N_phy
3	DIN	v	$\mu\text{gN/L}$	External dissolved inorganic N	DIN	DIN
4	r_max	p	1/day	Max phytoplankton growth rate	$r_{\{\text{max}\}}$	r_max
5	IN_max	p	$\text{gN/gC}$	Max internal N:C ratio	$IN_{\{\text{max}\}}$	IN_max
6	IN_min	p	$\text{gN/gC}$	Min internal N:C ratio	$IN_{\{\text{min}\}}$	IN_min
7	UN_max	p	$\text{gN/gC/day}$	Max N uptake rate	$UP_{\{\text{max}\}}$	UN_max
8	K_Nup	p	$\mu\text{gN/L}$	Half sat. for N uptake	$K_{\{\text{Nup}\}}$	K_Nup
9	uptake	f	$\mu\text{g}/\mu\text{gC/d}$	uptake rate for N	uptake	uptake
10	droop	f	$\mu\text{gC}/\mu\text{gC/d}$	realised growth rate of phytoplankton	droop	droop

# Processes

	<b>name</b>	<b>unit</b>	<b>description</b>	<b>expression</b>
1	Growth	μgC/d	Growth	$r\_max * droop(IN\_max, IN\_min, N\_phy/C\_phy)$
2	U_DIN	μgN/d	Uptake of DIN	$uptake(UN\_max, IN\_max, IN\_min, K\_Nup, DIN, N\_phy/C\_phy)$

# Stoichiometry

	<b>variable</b>	<b>process</b>	<b>expression</b>
1	C_phy	Growth	C_phy
2	N_phy	U_DIN	C_phy
3	DIN	U_DIN	-C_phy



## Define functions

```
uptake <- function(Umax, Qmax, Qmin, K_half, Ext, Int)
  {Umax*((Ext*(Qmax-Int))/((K_half+Ext)*(Qmax-Qmin)))}
```

```
droop <- function(Qmax, Qmin, Int)
  {(Qmax / (Qmax - Qmin)) * (1 - (Qmin / Int))}
```

# Neat documentation

# State variables

Symbol	Unit	Description
$C_{phy}$	$\mu\text{gC/L}$	Cellular C
$N_{phy}$	$\mu\text{gN/L}$	Cellular N
$DIN$	$\mu\text{gN/L}$	External dissolved inorganic N

# Parameters

Symbol	Unit	Description
$r_{max}$	1/day	Max phytoplankton growth rate
$IN_{max}$	gN/gC	Max internal N:C ratio
$IN_{min}$	gN/gC	Min internal N:C ratio
$UP_{max}$	gN/gC/day	Max N uptake rate
$K_{Nup}$	$\mu\text{gN/L}$	Half sat. for N uptake

# Code generation

## Instantiate model object

```
model <- new("rodeo", vars=subset(identifiers,type=="v")
            , pars=subset(identifiers,type=="p")
            , funs=subset(identifiers,type=="f")
            , pros=processes, stoi=stoichiometry)
```

# Generate code

## R or Fortran95

```
code <- model$generate(name="derivs",lang="r")  
derivs <- eval(parse(text=code))
```

## Provide parameter and starting values

```
pars <- list(r_max=1, K_Nup=100, UN_max=0.2
            , IN_max=0.13 * (14/12)
            , IN_min=0.05 * (14/12))

init_C_phy <- 2000
init_DIN <- 80

vars <- list(C_phy=init_C_phy, DIN=init_DIN
            , N_phy=(2*pars[["IN_min"]])*init_C_phy)

p <- model$arrangePars(pars)
v <- model$arrangeVars(vars)
```



## Test the derivative function

```
derivs(time, v, p, NLVL = 1)
```

```
## $dydt
```

```
## [1] 1625.00000    66.66667   -66.66667
```

```
##
```

```
## $pro
```

```
## [1] 0.81250000 0.03333333
```

# deSolve

# Solve the ODE system

## Pass parameters and starting values at runtime

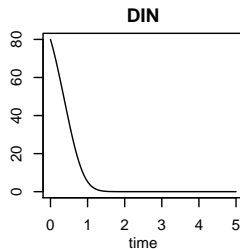
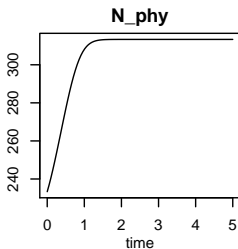
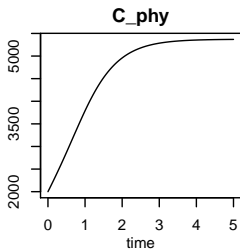
```
t <- seq(0, 5, length.out=100)

system.time({
  out_R1 <- ode(y=v, times=t, func=derivs
               , parms=p, NLVL=1)
})

##      user  system elapsed
##    0.06   0.00   0.08
```

Solve the ODE system

# Plot output



# Shiny!



## More ecology

- ▶ N **or** P limit growth rate
- ▶ Effect of light ( $I$ ) is multiplicative

$$\frac{dC_{phy}}{dt} = f(I) \cdot \min\{f(N), f(P)\}$$



# Shiny app demonstration



# Packages

- ▶ rodeo (David Kneis) <https://github.com/dkneis/rodeo>
  - ▶ Standard definition of ODE's in tables
  - ▶ Definition is the documentation
  - ▶ Code generation in R or F95 (250x faster)
- ▶ deSolve (Soetaert, Petzoldt, Setzer)
  - ▶ Numeric solvers for DE
- ▶ Shiny (Chang, Cheng, Allaire, Xie and McPherson)

## Acknowledgments

- ▶ David Kneis *rodeo* @TU Dresden
  - ▶ <https://github.com/dkneis/rodeo>
- ▶ Thomas Petzoldt @TU Dresden
- ▶ Sebastian Kolzau, Jacqueline Rücker, Claudia Wiedner @BTU
- ▶ Nitrolimit [www.nitrolimit.de](http://www.nitrolimit.de)
- ▶ BMBF FONA 033L041A and 0033W015AN



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## Petersen matrix

$$\begin{array}{l} \text{Growth} \\ \text{U DIN} \end{array}
 \begin{array}{c} C_{phy} \quad N_{phy} \quad DIN \\ \left( \begin{array}{ccc} C_{phy} & 0 & 0 \\ 0 & C_{phy} & -C_{phy} \end{array} \right) \end{array}
 \begin{array}{l} \text{Rate expressions} \\ \left( \begin{array}{l} f\left(\frac{N_{phy}}{C_{phy}}, IN_{max}, IN_{min}\right) \\ f\left(DIN, \frac{N_{phy}}{C_{phy}}, Nup_{max}, \dots\right) \end{array} \right) \end{array}$$

## Literature cited

- ▶ Davis, T.W., Bullerjahn, G.S., Tuttle, T., McKay, R.M. & Watson, S.B. (2015) Effects of Increasing Nitrogen and Phosphorus Concentrations on Phytoplankton Community Growth and Toxicity During Planktothrix Blooms in Sandusky Bay, Lake Erie. *Environmental Science & Technology*, 49, 7197–7207.
- ▶ Karline Soetaert, Thomas Petzoldt, R. Woodrow Setzer (2010). Solving Differential Equations in R: Package **deSolve** *Journal of Statistical Software*, 33(9), 1–25. URL <http://www.jstatsoft.org/v33/i09/>.

- ▶ Kolzau, S., Wiedner, C., Rucker, J., Köhler, J., Köhler, A. & Dolman, A.M. (2014) Seasonal patterns of nitrogen and phosphorus limitation in four German lakes and the predictability of limitation status from ambient nutrient concentrations. PLoS ONE, 9, e96065.